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Question Paper Code : X 20782

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020

Fourth Semester

Electronics and Communication Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Assume that X is a continuous random variable with the probability density function $f(x) = \begin{cases} \frac{3}{4}(2x - x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$. Find P(X > 1).

2. A random variable X is uniformly distributed between 3 and 15. Find the variance of X.

3. The equations of two regression lines of random variables X, Y are $4x - 5y + 33 = 0$, $20x - 9y - 107 = 0$. Find the mean values of X and Y.

4. Find the value of k if $f(x, y) = \begin{cases} kxye^{-(x^2+y^2)}; & x \geq 0, y \geq 0 \\ 0 \quad ; & \text{otherwise} \end{cases}$ is a joint probability density function of (X, Y).

5. Define a stationary process.

6. What is Markov process ?

7. State any two properties of cross correlation function.

8. Find the auto correlation function whose spectral density is $S(\omega) = \begin{cases} \pi, & |\omega| \leq 1 \\ 0, & \text{otherwise} \end{cases}$.

9. Check whether the system $y(t) = x^3(t)$ is linear or not.

10. If X(t) is a WSS process and if $y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$ then prove that $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$.

PART – B **(5×16=80 Marks)**

11. a) i) Find the mean and variance of the random variable X if the probability density function of X is given by $f(x) = \begin{cases} x: & 0 < x < 1 \\ 2 - x: & 1 < x < 2 \\ 0: & \text{otherwise} \end{cases}$. **(8)**

ii) Find the moment generating function of the Poisson distribution and hence find its mean and variance. **(8)**

(OR)

b) i) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probability that one of these tires will last at least 20,000 km. Also find the probability that one of these tires will last at most 30,000 km. **(8)**

ii) State and prove the memory less property of Geometric distribution. **(8)**

12. a) i) Let (x, y) be a two-dimensional non-negative continuous random variable having the joint density $f(x, y) = \begin{cases} 4xy\, e^{-(x^2 + y^2)}, & x \ge 0, y \ge 0 \\ 0, & \text{otherwise} \end{cases}$. Find the density function of $U = \sqrt{x^2 + y^2}$. **(8)**

ii) Given : **(8)**

$$f_{xy}(x, y) = \begin{cases} Cx(x - y), & 0 < x < 2, \quad -x < y < x \\ 0, & \text{otherwise} \end{cases}$$

1) Evaluate C
2) Find $f_x(x)$
3) $f_{y/x}(y/x)$ and
4) $f_y(y)$.

(OR)

b) i) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If x denotes the number of white balls drawn and y denotes the number of red balls drawn, find the joint probability distribution of (x, y). **(8)**

ii) In a partially destroyed laboratory record only the lines of regressions and variance of x are available. The regression equations are $8x - 10y + 66 = 0$ and $40x - 18y = 214$ and variance of x = 9. Find :
1) The correlation coefficient between x and y
2) Mean values of x and y
3) Variance of y. **(8)**

13. a) i) A random process {X(t)} is defined by X(t) = A cos t + B sin t, $-\infty < t < \infty$ where A and B are independent random variables each of which has a value –2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. Show that {X(t)} is a wide sense stationary process. **(8)**

ii) Suppose the customer arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes.

1) Exactly four customers arrive.
2) Greater than 4 customers arrive.
3) Fewer than 4 customers arrive. **(8)**

(OR)

b) i) A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train. But he drives one day, then the next day is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes a train on the fourth day and the probability that he drives to work on the fifth day. **(8)**

ii) Define semi random telegraph signal process and prove that it is an evolutionary process. **(8)**

14. a) i) Consider two random processes $X(t) = 3\cos(\omega t + \theta)$ and $Y(t) = 2\cos(\omega t + \phi)$, where $\phi = \theta - \frac{\pi}{2}$ and θ is uniformly distributed over $(0, 2\pi)$. Verify $|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)\, R_{YY}(0)}$. **(8)**

ii) Find the power spectral density of a random binary transmission process where auto correlation function is $R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & : |\tau| \le T \end{cases}$. **(8)**

(OR)

b) i) If the power spectral density of a continuous process is

$S_{XX}(\omega) = \frac{\omega^2 + 9}{\omega^4 + 5\omega^2 + 4}$, find the mean square value of the process. **(8)**

ii) A stationary process has an auto correlation function given by

$R(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$. Find the mean value, mean-square value and variance of the process. **(8)**

15. a) i) If a system is connected by a convolution integral $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$ where X(t) is the input Y(t) is the output then prove that the system is a linear time invariant system. **(8)**

ii) X(t) is the input voltage to a circuit and Y(t) is the output voltage. {X(t)} is a stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{2|\tau|}$. Find μ_y, $S_{YY}(\omega)$ and $R_{YY}(\tau)$, if the system function is given by $H(\omega) = \frac{1}{\omega + i^2}$. **(8)**

(OR)

b) i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ and $S_{YY}(\omega)$ are the power spectral density functions of the input X(t) and the output Y(t) respectively and $H(\omega)$ is the system transfer function. **(8)**

ii) A linear system is described by the impulse response $h(t) = \left(\frac{1}{RC} e^{-t/RC}\right) u(t)$. Assume an input process whose auto correlation function is $A\delta(\tau)$. Find the mean and auto correlation function of the output process. **(8)**